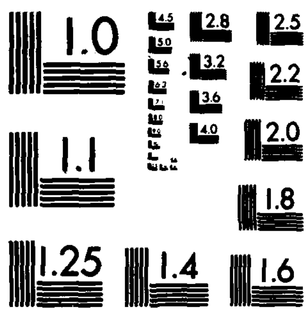
AD-A116 128 TELEDYNE GEOTECH ALEXANDRIA VA SEISMIC DATA ANALYSIS--ETC F/G 8/11
EVALUATION OF SEISMIC EVENT LOCATION METHODS ON THE REGIONAL EV--ETC(U)
NOV 81 A C CHANG, D W RIVERS, J H GONCZ F08606-79-C-0007
UNCLASSIFIED SDAC-TR-81-10 VSC-TR-82-8 NL

END
DATE
FILMED
8-82
DTIC



MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER VSC-TR-82-8	2. GOVT ACCESSION NO. AD-A116 128	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) EVALUATION OF SEISMIC EVENT LOCATION METHODS ON THE REGIONAL EVENT LOCATION SYSTEM (RELS)		5. TYPE OF REPORT & PERIOD COVERED Technical
		6. PERFORMING ORG. REPORT NUMBER SDAC-TR-81-10
7. AUTHOR(s) A. C. Chang, D. W. Rivers, J. H. Goncz, J. A. Burnetti		8. CONTRACT OR GRANT NUMBER(s) F08606-79-C-0007
9. PERFORMING ORGANIZATION NAME AND ADDRESS Teledyne Geotech 314 Montgomery Street Alexandria, Virginia 22314		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS VT/0709
11. CONTROLLING OFFICE NAME AND ADDRESS VELA Seismological Center 312 Montgomery Street Alexandria, Virginia 22314		12. REPORT DATE 11/26/81
		13. NUMBER OF PAGES 51
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) Defence Advanced Research Projects Agency 1400 Wilson Boulevard Arlington, Virginia 22209		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Location Methods
RELS Location
Location Accuracies
Locations with Regional Models
Locations with Correlation Matrix
Locations with Regional Data
Location Confidence Ellipses

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Several location methods developed at SDAC are evaluated by comparing location errors of these methods against errors computed with the standard method. The new methods include:

- Using P travel times in laterally heterogeneous media;
- A station travel time residual correlation matrix in the normal equations;
- Location with simultaneous determination of P_n, P_g and L_g velocities;
- Combination of the correlation matrix method and the simultaneous determination method.

DD FORM 1473, 1 JAN 73 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

All the methods evaluated in this report appear to give smaller absolute epicenter and depth errors than the standard method of locating events. Furthermore, these methods do not use station corrections; therefore the locations obtained are not station- or region-dependent. They can be used to locate seismic events in any source region, with any set of stations.

When average location errors are compared against errors with the standard method, the correlation matrix method is about 3 km better, e.g., about 17 kilometers as compared to 20 kilometers; the regional models method is 3 to 6 km better, e.g., about 5 kilometers as compared to 9 kilometers, depending on whether near-regional stations are used. The average location errors from the true epicenters using regional data are approximately 6 km.

The addition of L_g arrivals to locate events did not result in better locations. Typically, we added 6 L_g arrival times to a total of 15 P_n and P_g arrival times. The lack of improvement may be attributed to a greater variance in L_g travel times.

When events were located with both teleseismic P and regional phases, the weighting factors for these phases played an important role. When all phases were weighted equally, location errors were smaller than when the P phases were weighted more heavily. Depth errors using all phases were similar to those computed with teleseismic P arrivals only. This is to be expected because the travel times of regional phases are affected weakly, if at all, by event depth.

The six most thoroughly analyzed events were FAULTLESS, RULISON, SHOAL, GASBUGGY, SALMON, and GNOME. The median number of P arrivals was74; of P_n, 10; of P_g, 1; and of L_g, 4. The median location error using the standard method was about 20 kilometers and the median depth error was also about 20 kilometers. By adding the regional phase arrivals and weighting each phase equally, these numbers were reduced to 3.6 and 3.7 kilometers respectively. Furthermore, while the true epicenter lay outside the 95% confidence ellipse 5 out of 6 times in the standard case, it lay within the ellipse in all cases in the improved analyses. These results represent a remarkable improvement in absolute location accuracy. Most of the improvement is due to the use of regional phases.

EVALUATION OF SEISMIC EVENT LOCATION METHODS
ON THE REGIONAL EVENT LOCATION SYSTEM (RELS)

SEISMIC DATA ANALYSIS CENTER REPORT NO.: SDAC-TR-81-10

AFTAC Project Authorization No.:	VELA T/0709/B/PMP
Project Title:	Seismic Data Analysis Center
ARPA Order No.:	2551
Name of Contractor:	TELEDYNE GEOTECH
Contract No.:	F08606-79-C-0007
Date of Contract:	01 October 1980
Amount of Contract:	$1,493,393
Contract Expiration Date:	30 September 1981
Project Manager:	Robert R. Blandford (703) 836-3882

P. O. Box 334, Alexandria, Virginia 22313

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION UNLIMITED.

ABSTRACT

Several location methods developed at SDAC are evaluated by comparing location errors of these methods against errors computed with the standard method. The new methods include :

Using P travel times in laterally heterogeneous media;

A station travel time residual correlation matrix in the normal equations;

Location with simultaneous determination of Pn, Pg and Lg velocities;

Combination of the correlation matrix method and the simultaneous determination method.

All the methods evaluated in this report appear to give smaller absolute epicenter and depth errors than the standard method of locating events. Furthermore, these methods do not use station corrections; therefore the locations obtained are not station- or region-dependent. They can be used to locate seismic events in any source region, with any set of stations.

When average location errors are compared against errors with the standard method, the correlation matrix method is about 3 km better, e.g., about 17 kilometers as compared to 20 kilometers; the regional models method is 3 to 6 km better, e.g., about 5 kilometers as compared to 9 kilometers. depending on whether near-regional stations are used. The average location errors from the true epicenters using regional data are approximately 6 km.

The addition of Lg arrivals to locate events did not result in better locations. Typically, we added 6 Lg arrival times to a total of 15 Pn and Pg arrival times. The lack of improvement may be attributed

to a greater variance in Lg travel times.

When events were located with both teleseismic P and regional phases, the weighting factors for these phases played an important role. When all phases were weighted equally, location errors were smaller than when the P phases were weighted more heavily. Depth errors using all phases were similar to those computed with teleseismic P arrivals only. This is to be expected because the travel times of regional phases are affected weakly, if at all, by event depth.

The six most thoroughly analyzed events were FAULTLESS, RULISON, SHOAL, GASBUGGY, SALMON, and GNOME. The median number of P arrivals was 74; of Pn, 10; of Pg, 1; and of Lg, 4. The median location error using the standard method was about 20 kilometers and the median depth error was also about 20 kilometers. By adding the regional phase arrivals and weighting each phase equally, these numbers were reduced to 3.6 and 3.7 kilometers respectively. Furthermore, while the true epicenter lay outside the 95% confidence ellipse 5 out of 6 times in the standard case, it lay within the ellipse in all cases in the improved analyses. These results represent a remarkable improvement in absolute location accuracy. Most of the improvement is due to the use of regional phases.

TABLE OF CONTENTS

LIST OF TABLES

1. INTRODUCTION

For the past several years, we have been investigating various methods for improving seismic location estimates. Improvements can be made when the Earth's lateral heterogeneities are properly handled. However, this is not a simple task because large fluctuations in layer velocities are observed over very small distances.

Seismic events are traditionally located at teleseismic distances with teleseismic P arrivals. The use of later phases is somewhat restricted for various reasons. For example, PKP arrivals are often used together with P, but only one branch, DF, has typically been used in location programs. On the other hand, the BC branch has the greatest amplitudes, and BC arrivals are often observed instead of DF arrivals. pP arrivals are very useful in identifying the event depth, but observation of pP is often not easy because of the presence of the coda. We have developed a program which has two distinct options for locating seismic events using teleseismic P. The first option computes source to station travel times by using a priori knowledge of regional Earth models. The second option uses a correlation matrix in the normal equations in which the matrix elements are calculated from a simple function of inter-station distance.

Studies in recent years have shown that the crust of the Earth is probably more heterogeneous than we previously thought. Lateral changes in layer velocities can be on the order of 0.1 km/sec instead of 0.01 km/sec. Under such conditions, it can be very difficult to obtain accurate location estimates using Pn and Pg arrivals. Although some workers have shown that accurate locations can be obtained by using three-dimensional ray-tracing programs to compute travel times, such methods require exact knowledge of the structure of the region. Thus,these methods are not applicable to an area of unknown geologic structures, and seismic event location in such areas is often necessary. Our investigations in seismic location using Pn and Pg resulted in two possible methods. Both begin the location with a simple standard Earth model and modify layer velocities as the location proceeds. The methods determine layer velocities by either successive determinations (SUCCESS) or simultaneously with seismic locations (SIMUL), and both give much better results than the traditional location method.

These studies have resulted in some useful options for location programs. However, these options had not been collected in a program which could easily use them all. Our objective was to create a simplified location program with all the available options combined. This program was developed on the VAX 11/780 for the Regional Event Location System (RELS). This report describes the program developments in RELS, and some results of an evaluation of the various location methods developed in the previous years.

2. PROGRAM DEVELOPMENTS

Conversion of the location program from the TS44 system to the RELS on VAX 11/780 seemed to be a straightforward task, but it took longer than expected. Part of the reason was a lack of programming support and some delays in system development. However, some significant changes in the location program were made during this period. The following highlights some of these changes.

2.1. Options in the Location Program

Four options in the location program were developed in the past. They are:

(1) ACLOC: Locations using models of the laterally heterogeneous crust and upper mantle. Travel times were not computed by ray tracing, but by adding travel time corrections to the Herrin ´68 table. These corrections are stored in disk area REGICO on TS44. This capability has not yet been transferred to the VAX 11/780, as discussed below.

(2) LOCATION: This progam computes locations with the addition of a correlation matrix.

(3) SUCCESS: Location method using Pn and Pg. Crustal layer velocities are modified to improve the location estimates by successive determinations.

(4) SIMUL: Location method using Pn and Pg. Crustal layer velocities are determined at the same time locations are computed. This

option also allows locations using azimuth information only.

ACLOC requires an extra disk area called REGICO. Since the allocation of disk areas on the VAX was not determined at the time we wished to move these programs. the conversion of ACLOC to the VAX was delayed until some later date. We developed SUCCESS and SIMUL to locate events with Pn and Pg. The option SIMUL also contains the option to compute locations with azimuth information obtained from the Smart processor. Therefore, two of the options, LOCATION and SIMUL, were converted and merged. (The conversion of SUCCESS was delayed for the same reasons as the conversion of ACLOC.)

2.2. Formation of Modules

Since all of the subroutines and most parts of the main routine in each of these different options are identical, merging the options to form a single program was highly desirable. This was not possible on the TS44 system, because all the options were stored as independent programs. The convenient file structure on the VAX enabled such a merger to be easily accomplished. First, the main subroutine LOK in both options was divided into several sections, so that no duplicate coding was needed. Second, only those portions with different coding were filed under separate names. Third, a driver routine was created to pick up various modules according to the option desired for execution. This method also enables us to add more options in the future.

2.3. Directory of Station Coordinates

The directory of seismic stations and station coordinates containing approximately 2950 stations is called STATCO, and it is stored in the TS44SOR disk area. Geographic coordinates of stations in degrees, minutes, and tenths of seconds are stored in this file. Additional information includes geographic and geocentric co-latitudes and east longitudes of stations in integer format, elevation and alphanumeric station information. This file has been in use at SDAC for many years. However, updating station coordinates has not been done for some time. For example. station names and coordinates of Alaskan sites, SRO and ASRO stations, etc., are not stored in STATCO.

The station coordinate file on the VAX is called STATCOORD.DAT. This file contains station co-latitude and longitude in radians, and some information about the status, history, operating conditions,and elevation of the stations. More than 3500 stations are in the file.

After making some format changes to accommodate the new station coordinate files, we have compared some station coordinates in both systems. Some discrepancies were found in the deci-second field. However, these discrepancies were all traced to errors in STATCO on the TS44 system.

2.4. Arrival Time Conversions

The arrival time inputs in the location program are formatted in integers of hour, minute, and deci-seconds. Upon converting them into floating point values in seconds, some discrepancies at the deci-second level were found between the TS44 version and the VAX version of location programs. However, these errors do not occur all the time. The error was caused by the fact that the IBM computer's floating point format does not have enough bits to contain all 24 hours of time in seconds. Consequently, the least significant bit would be lost on some occasions in which the time of the day was close to the end of the day. The VAX version of time conversion has not displayed this type of problem.

2.5. Matrix Inversion Routine

Matrix inversion requires a large memory area and a long computing time. It was not possible to install an accurate matrix inversion routine in the TS44 system because of its core and computation time limitations.

The location method with correlation matrix was worked out in a previous paper (Chang et al. 1980). However, it was not possible to compute an accurate inverse matrix at that time. The result of locations with a correlation matrix was consequently tested with only one event, LONGSHOT. A total of 184 station inputs was divided into 9 station groups, and inversions were carried out in the corresponding block diagonal form.

A robust matrix inversion routine was adopted from the IMSL library and installed in the VAX location program. This is a significant achievement in that we can now use the location with the correlation matrix option for any event without station groupings. A test run of the LONGSHOT event with a 186 by 186 matrix was successful. Since the location program is currently limited to a maximum of 200 station inputs, we think the test shows that the program can satisfactorily handle the maximum input case. The VAX takes approximately 2.5 minutes of CPU time to invert a matrix of this size.

2.6. Distance Azimuth Computation

In the past, source-to-station distance and azimuth computations have been carried out in single precision. Single precision computation is satisfactory in most cases, but we found that computations in double precision were necessary when two stations were near each other. It was necessary to determine the effect of this error on the location program.

The subroutine DIAZ in the ADAPS system was adopted and converted to a subroutine with double precision computations. Distance computations between two stations have been compared with the existing subroutine BJDAZ and the newly created double precision subroutine ACDIAZ. It was found that differences are very small in most cases; all were within 50 meters of distances computed with the double precision routine.

3. EVALUATION OF TELESEISMIC LOCATION METHODS

3.1. Seismic Event Locations In Laterally Heterogeneous Earth

Geiger's least squares method of locating seismic events determines the event location by minimizing the travel time residuals, i.e., the differences of observed and theoretical travel times computed with a standard Earth model. However, the Earth is in fact not laterally homogeneous. Therefore, locations computed by a least squares fit to a laterally homogeneous Earth model have errors due to the Earth's heterogeneities. The amount of error will depend on the distribution of stations and on the deviations of true Earth structures in those station areas. Traditionally, station travel time corrections are added to compensate for such errors. But it has been found that a simple constant correction for each station may not be sufficient for waves emerging from all directions.

A simple method of computing travel times for a laterally heterogeneous Earth model for each source-to-station path was developed by Chang et al. (1980). In this method, called ABSLOC, halfway travel times of the rays computed with Herrin 1968 model for various distance ranges i and depth d, T(H,d,i), are computed and subtracted from the halfway travel times computed for other regional Earth models A, B, C, D, E, etc., thus resulting in tables of halfway travel time corrections for those regional models with respect to the Herrin model.

$$\delta t(A,d,i) = T(A,d,i) - T(H,d,i) \qquad (1)$$

These halfway travel time corrections are stored in the computer in tabular form. One can obtain halfway travel times of any model simply by adding these corrections to the halfway travel time of the Herrin '68 model, i.e.,

$$T(A,d,i) = T(H,d,i) + \delta t(A,d,i) \tag{2}$$

The travel time of a laterally heterogeneous Earth, starting from the source in region A and emerging at a station in region B, is equal to the standard Herrin travel time for distance i plus travel time corrections for regional models A and B,

$$\begin{aligned} T(A,B,d,i) &= T(H,d,i) + \delta t(A,d,i) + T(H,0,i) + \delta t(B,0,i) \\ &= TH(d,i) + \delta t(A,d,i) + \delta t(B,0,i) \end{aligned} \tag{3}$$

where TH(d,i) is the standard Herrin travel time at source depth d and distance i. Seismic event location with ray paths through a laterally heterogeneous Earth is made possible in this manner. A test of this method using four nuclear explosions showed that errors were reduced to about half of the errors computed with standard methods (loc. cit.). However, it was necessary to assign regional models manually for each source-to-station path; furthermore, only four regional models were used.

In the subsequent year, Chang et al. (1981a) modified the method so that regional models for source and station areas can be assigned without being input manually by the analyst. This new method, called ACLOC, uses the 729 geographic regions of Flinn et al. (1974), and one regional model is assigned to each geographic region. A total of 20 regional models was adopted from the published literature and computations were made for travel time corrections. However, almost half of these models were found to be inadequate because the resulting travel time corrections did not agree with published travel time residuals. By eliminating the inadequate models, a total of 11 regional models was left in the ACLOC method.

The effectiveness of the ACLOC method was compared against the traditional location method using Herrin '68 travel times. A total of 12 nuclear explosions were used for this comparison, and location errors of these methods, tested for depth-free and depth-restricted runs, are shown in Table I. Results in Table I show that the ACLOC method gives better location estimates than HERRIN, in either depth-free or depth-restricted runs. On the average, the location errors are reduced by about 3 kilometers.

One reason that greater improvements in errors were not observed is that there were many seismic stations that lie closer than 30 degrees from the event. Seismic rays travelling through the Earth for these regional stations penetrate only to about 800 kilometers, depending on the distance from the source. Since the Earth's crust and upper mantle are known to be quite heterogeneous, these stations are the ones that are most influenced by incorrect regional models. Subsequent tests, using only stations at distances greater than 24 degrees, resulted in slightly better locations. These results are shown in Table II.

The ACLOC method should give better results because it corrects for lateral heterogeneities. The method is limited by our lack of knowledge of the Earth's heterogeneity, which appears in two ways. The first is that the models we choose are not adequate; and secondly, the selection of the model assigned to some of the areas may not be adequate. The adequacy of the assigned models was examined by comparing travel times computed through them to published travel time residuals. Further, one can calibrate some of the model assignments by comparing the published residuals of some stations against the corrections assigned by the computer. This was done by checking the residuals of the ACLOC runs and re-assigning regional models to those stations that had high residuals. The calibrated version is called ACLOC3.

In addition to the travel time residuals data published by Sengupta and Julian (1974) and by Cleary and Hales (1966), we acquired a copy of the worldwide travel time residuals published by Poupinet (1979). This latter paper covers more worldwide areas than the previous papers. Thus, another version called ACLOC4, with regional model assignments based on Poupinet's residuals, was created. Table III compares the location errors of these various versions, HERRIN, ACLOC, ACLOC3, and ACLOC4. The results of ACLOC4 are about the same as those of ACLOC; and the results of ACLOC3, which used the residuals of Sengupta and Julian (1974) and of Cleary and Hales (1966), are the best among these versions. It is, however, possible that ACLOC4 will give better locations when the model assignments are corrected for bad stations. The average improvement in location errors as compared to HERRIN in Table III is about 6 kilometers,declining from 21 to 15 kilometers.

TABLE I

Comparison of Location Errors, ACLOC vs. HERRIN
(Stations within 12 degrees are excluded)

Event Name	No. Sta.	Location Errors in Kilometers HERRIN-R	ACLOC-R	HERRIN-F	ACLOC-F
LONGSHOT	184	11.04	7.70	19.42	16.60
GNOME	44	43.83	45.68	43.83	46.75
SHOAL	21	51.64	24.88	41.52	44.07
SALMON	75	22.91	16.36	20.73	16.15
CORDUROY	89	12.19	20.08	20.93	20.80
GASBUGGY	60	21.19	9.00	31.30	22.18
FAULTLESS	106	6.98	7.46	5.46	2.46
PURSE	121	9.77	9.27	14.89	12.45
RULISON	74	12.88	7.08	15.51	10.64
PIPKIN	103	10.32	7.76	26.05	22.96
TERRINE	53	6.03	12.44	24.05	16.44
CARPETBAG	136	7.09	8.54	11.66	7.41
Average		17.99	14.69	22.95	19.91

R - depth restricted, F - depth free

TABLE II

Comparison of Location Errors, ACLOC vs. HERRIN
(Stations within 24 degrees are excluded)

Event Name	No. Sta.	Location Errors in Kilometers HERRIN-R	ACLOC-R	HERRIN-F	ACLOC-F
LONGSHOT	168	21.85	18.05	21.32	17.10
GNOME	11	15.93	34.47	56.31	59.48
SHOAL	10	16.66	16.56	42.29	44.81
SALMON	26	17.64	8.49	14.68	7.58
CORDUROY	72	9.64	6.56	8.88	7.39
GASBUGGY	40	22.43	21.74	31.77	27.63
FAULTLESS	101	5.58	2.01	4.66	2.19
PURSE	108	13.98	11.76	16.88	13.07
RULISON	55	1.86	4.09	6.51	4.19
PIPKIN	85	16.57	15.41	20.21	15.26
TERRINE	42	16.92	15.81	19.08	13.66
CARPETBAG	118	15.32	13.12	15.80	13.30
Average		14.53	14.00	21.53	18.81

R - depth restricted, F - depth free

A comparison of the depth errors for various methods is also shown in Table III. On the average we find that depth errors are two to three times larger than location errors; this is a well known phenomenon for shallow events. If we compare the average depth errors of HERRIN to those of ACLOC or ACLOC3, we see that depth errors also are reduced in the ACLOC method. However, without accurate knowledge of regional geologic structures, further improvements of the ACLOC method are rather difficult.

It is noteworthy that all depths computed with the HERRIN method are positive, while ACLOC and ACLOC3 methods resulted in some negative depths. We also note that those events with negative depths are all at NTS. Since the mean depth over an ensemble of events at NTS should be near zero, or perhaps 1 km, this suggests that the source model assignments for NTS may be proper.

TABLE III

Comparison of Three Versions of ACLOC vs. HERRIN
(Stations within 24 degrees are excluded, all runs depth free)

Event Name	Location and Depth Errors in Kilometers			
	HERRIN (d)	ACLOC (d)	ACLOC3 (d)	ACLOC4 (d)
LONGSHOT	21.32 (98.9)	17.10 (88.0)	13.53 (92.0)	14.94 (77.2)
GNOME	56.31 (89.4)	59.48 (37.8)	39.41 (37.8)	58.30 (80.6)
SHOAL	42.29(165.2)	44.81(158.8)	30.83(158.8)	46.92(163.7)
SALMON	14.68 (35.4)	7.58 (12.6)	12.95 (12.6)	7.03 (35.9)
CORDUROY	8.88 (27.8)	7.39(-13.3)	10.57(-18.9)	9.96 (35.8)
GASBUGGY	31.77 (88.2)	27.63 (55.8)	19.71 (50.2)	24.15 (84.5)
FAULTLESS	4.66 (33.7)	2.19(-24.4)	3.50 (53.7)	6.71 (30.7)
PURSE	16.88 (57.5)	13.07 (28.2)	7.77 (28.7)	13.98 (54.3)
RULISON	6.51 (73.9)	4.19 (38.3)	4.27 (41.9)	8.42 (71.6)
PIPKIN	21.21 (36.6)	15.26 (-1.7)	17.28 (-2.4)	20.57 (37.2)
TERRINE	19.08 (21.8)	13.66(-23.2)	11.16(-24.3)	12.00 (13.3)
CARPETBAG	15.80 (51.1)	13.30 (23.3)	6.92 (23.0)	12.77 (50.0)
Average	21.53 (65.0)	18.81 (42.1)	14.83 (43.4)	19.65 (61.2)

* Average depth errors are the average of absolute values.

3.2. Locations With Correlation Matrix

The ACLOC method discussed in the previous section assumes that travel time residuals are fixed deterministic quantities that can be independently estimated for each source and station. Alternately, one can eliminate the location bias due to clusters of seismic station located in a close proximity by weighting.

The second method of locating teleseismic events, LOCATION, assumes that stations in close proximity may be measuring the same effect of regional structure. Thus by inserting a correlation matrix in the normal equation, the effect of clustered stations will be corrected by the weighting of the travel time residuals in accordance with their overall intercorrelation matrix. The advantage of this method is that the correction of station bias improves not only the location estimates, but also the confidence intervals, which will not contract substantially as arrival times from highly correlated stations are added to the data base.

In the previous work, correlation coefficients of residuals were estimated by using 186 station residuals of LONGSHOT event (Chang et al. 1980). Coefficients of 0.7 for $0.1° < \Delta < 5°$, and 0.3 for $5° < \Delta < 10°$ were used to test the method. However, the method was not thoroughly tested because an accurate matrix inversion routine was not available on the TS44 system. The new version of LOCATION on VAX is capable of fully testing this method.

The correlation coefficients on the VAX location program are computed with the following equations:

$$c = 0.7\ \exp(-(\Delta/7^\circ)^2)\ ; \quad \text{for } \Delta > 0 \text{ degree, and}$$

$$c = 1.0 \qquad ; \quad \text{for } \Delta = 0 \text{ degree.}$$

Values computed with this equation are in agreement with those used in the previous work, but are better in the sense that the values are continuous with distance. However, correlation structures should be further investigated in the future. The question of whether these correlation coefficients vary from one region to another is also an important matter to be determined in the future.

A total of 15 nuclear explosions were used to evaluate the LOCATION method of locating events with correlation matrix. These 15 events include 12 explosions used in Tables I - III, plus two French Sahara tests and a Kazakh event. The location coordinates of the French tests were given by Bolt (1976). The Kazakh event was a cratering event, and the location of the crater was determined from a satellite photograph (Fitch and North, 1980).

Comparison of location errors using a standard location method with the Herrin table are compared with errors using the correlation matrix. The result shows that better locations can be obtained using the correlation matrix method. However, improvements are not as great as for the LONGSHOT event in most cases. The reason for this is that for some events like LONGSHOT, many stations were situated in close proximity; thus, for these events. station bias is removed by using the correlation matrix method. For the other events, stations are not distributed in close proximity to each other, thus stations exerting regional biases are not overweighted, and there are no appreciable improvements. One can compare these events in Table IV with those corresponding events with the ACLOC method in Table I, and find that the the improvement is about the same. Note that both methods are better than standard location results.

The Kazakh event is the only event for which SRST corrections are available (Veith, 1974). We have relocated this event in two ways: with the SRST corrections, and without them. Locations were sightly better when the SRST corrections were applied to the HERRIN location method, but no improvements were observed when SRST corrections were used together with the correlation matrix method. In the latter case, the lack of improvement using SRSTs may be attributable to the fact that a number of stations used for the Kazakh event were not included in the SRST corrections table.

Absolute depth errors for the LOCATION method are about the same as depth errors for the ACLOC method. The advantage of the LOCATION method over the ACLOC method is that, without knowing the details of the biasing geology, it minimizes the effects of differences between ray paths in the upper mantle. This allows the LOCATION method to use all the station inputs. However, if we compare the results of ACLOC methods using all stations (Table I), we find that the difference in location errors are very small.

If the station bias is properly removed by the LOCATION method, the shape of the resulting error ellipse should also be more symmetrical. Removal of bias will also increase the chance of containing the true location within the computed error ellipse. Test results in Table IV confirm this. In spite of only slight improvement in location errors, more events relocated by the LOCATION method resulted in error ellipses including the true location. In HERRIN-F runs, only two out of 16 events contain the true locations within the computed error ellipses. For two of the remaining events, the true locations lay outside of the three-dimensional ellipsoids, but within the two-dimensional ellipses; and for twelve events, the true locations lay outside of both the three- and two-dimensional error ellipses. In contrast, there were only four out of 16 events in LOCATION-F runs for which two-dimensional error ellipses did not contain the true locations, and five events where the true locations lie outside both the three- and two-dimensional ellipses. One event (RULISON) resulted in a true location inside the three-dimensional ellipsoid, but outside the two-dimensional error ellipse. This unusual result will be discussed in the appendix.

Better epicenters were obtained when event depths were restricted, with only one event in HERRIN-R runs not containing the true location in the error ellipse. For three out of six LOCATION-R runs, the true locations were inside the error ellipses.

TABLE IV

Comparison of Location Errors, LOCATION vs. HERRIN
(Stations within 12 degrees are excluded)

Event Name	Location and Depth Errors in Kilometers HERRIN-R	LOCATION-R	HERRIN-F	(d)	LOCATION-F	(d)
LONGSHOT	&11.29	4.90	#19.51	(82.0)	*10.76	(65.2)
GNOME	&43.78	&42.00	#43.78	(-0.6)	#42.07	(3.1)
SHOAL	51.73	37.01	*41.44	(115.4)	41.17	(109.1)
SALMON	&22.81	12.97	#20.68	(20.3)	12.42	(-23.7)
CORDUROY	12.31	1.57	#21.03	(72.6)	#23.08	(80.0)
GASBUGGY	&21.46	&30.68	#31.51	(40.5)	#35.12	(32.7)
FAULTLESS	7.03	5.54	* 5.46	(49.3)	*5.92	(55.1)
PURSE	& 9.94	7.37	#14.97	(56.5)	#15.19	(58.7)
RULISON	&13.11	&14.49	#15.66	(22.4)	$15.78	(13.2)
PIPKIN	10.38	16.83	#26.05	(67.6)	#30.26	(69.6)
TERRINE	5.80	4.71	#24.21	(64.7)	*23.52	(68.9)
CARPETBAG	7.18	6.87	#11.73	(61.6)	*12.80	(57.3)
SAPHIR	2.00	2.39	2.54	(-4.1)	2.53	(3.3)
RUBIS	9.11	6.61	8.49	(-5.8)	5.94	(-6.4)
KAZAKH	&10.55	5.60	#10.49	(0.9)	5.45	(2.9)
KAZAKH(SRST)	& 6.79	5.71	# 8.13	(-17.0)	6.52	(-15.3)
Average	15.33	12.83	19.11	(42.6)	18.03	(41.5)

R - Depth restricted

F - Depth free

* - True location lies outside of the 3D 95% F-statistic ellipse

- True location lies outside of the 3D and 2D 95% F-statistic ellipses

& - True location lies outside of the 2D 95% F-statistic ellipse

$ - True location lies inside of the 3D, but outside of the 2D 95% F-statistic ellipses.

4. SEISMIC EVENT LOCATIONS WITH REGIONAL DATA

4.1. Locations with Pn, Pg and Lg

The standard method of locating events, at both regional and teleseismic distances, computes event locations with travel times computed from a standard earth model. Because the layer velocities of the crust and upper mantle can change in a rather short distance, location accuracies using Pn and Pg may vary from region to region. It is known that better locations may be obtained if an adequate regional model is used, but such a regional model is not available for most areas.

Methods developed in the previous year are designed to compensate for such local variations by modifying layer velocities while computing event locations. Details of these methods are discussed in Chang et al. (1981b).

On the VAX version of the simultaneous inversions method (SIMUL), the capability of using Lg arrivals was added to the capability of using Pn and Pg. The Lg phase is assumed to have an initial velocity of 3.54 km/sec, and this value is modified during the iteration cycle. It is normally difficult to use Lg for locating events because its velocity is not well determined. However, the SIMUL method can overcome this difficulty by iterative modifications.

We will now compare the location errors using Pn, Pg (SIMUL) with the errors from using Pn, Pg and Lg (PLUSLG) in application to data from 12 explosions.

The location program on the VAX was programmed with Julian's (1973) original idea of weighing each arrival datum. In order to use all the regional data for each event in a single location, errors of observation of 1.0 second for Pn and 3.0 seconds for Pg and Lg are adopted from McCowan and Needham (1978). The intended effect of this was to reduce the dependence of the epicenter on those arrival times which cannot be read as accurately as Pn. Each observation (diagonal element of the covariance matrix) is multiplied by the inverse of these error values, so that Pg and Lg are weighted as only one third of the Pn observation. However, we are not certain that these error values are suitable for general use. In order to show the effect of weighting, the 12 explosions were re-located in two ways; one as above, and the other weighting all phases equally. Comparison of the location errors are shown in Table V.

Many interesting conclusions can be drawn by examining the results in Table V. These are:

(1) The addition of Lg arrivals in locating events did not improve the locations. However, no significant deterioration is seen either. The usefulness of Lg phase in location projects depends on how accurately these arrival times can be determined, and whether there are azimuthal variations in Lg velocities.

(2) Changing the weighting factors did not change the location errors appreciably. However, changing these weighting factors to some other values, for example, 1.5 for Pg and 3.0 for Lg, may be tried in the future.

(3) Table V shows 12 events which were relocated four different ways. For each of the four experiments location errors are mostly within 10 kilometers of the true location. In each experiment the average error is about 6 kilometers. This result is significantly better than location errors with using teleseismic data, as shown in Tables I through IV.

(4) Although there are a number of input data in each event, actually there were only a few stations used in these events. By examining the number of Pn, Pg and Lg used in each case, one finds that the number of stations in these events ranges from 4 to 13. Although this number may be typical for most seismic events, we point out that no appreciable improvements in Pn, Pg and Lg velocities can be made with only four or five stations. With this small number of stations it is remarkable that location errors with regional data are much better than teleseismic locations.

(5) With the exception of GASBUGGY, better locations can be obtained with an increased number of stations.

(6) Examination of residuals in GASBUGGY shows that 4 out of 12 Pg inputs show large residuals, and 2 out of 6 Lg residuals are also high. Two bad cases of Lg residuals suggest that Lg velocities may be different in different directions of propagation. However, it is interesting to note that although bad residuals are observed in Pg and Lg arrivals, location errors are better when all phases are weighted equally.

TABLE V

Comparison of Location Errors, SIMUL vs. PLUSLG
(Depth restricted)

Event Name	No. Pn	No. Pg	No. Lg	Unequal Weights SIMUL	Unequal Weights PLUSLG	Equal Weights SIMUL	Equal Weights PLUSLG
FAULTLESS	5	5	5	*18.65	11.99	7.37	1.19
RULISON	10	7	4	0.69	0.75	5.11	3.48
PASSAIC	4	3	4	3.04	3.07	3.49	* 3.89
ROCKVILLE DAM	9	8	8	4.43	3.78	16.02	*10.69
DORMOUSE PRIME	3	4	4	7.00	7.84	10.21	8.58
KLICKITAT	13	12	12	0.76	1.18	0.35	1.49
BANDICOOT	5	4	5	8.66	2.23	2.23	6.50
SHOAL	12	0	4	1.88	1.49	1.88	4.31
MERRIMAC	9	0	8	5.77	7.11	5.77	14.57
GASBUGGY	12	12	6	*10.03	* 9.68	* 8.92	* 8.54
PILEDRIVER	5	5	2	5.73	5.73	4.53	4.53
ROANOKE	4	4	4	4.10	7.05	5.08	*10.28
Average				5.90	5.16	5.91	6.50

* - True location lies outside of the 2D 95% F-statistic ellipse

4.2. Location with All Phases

So far we have been evaluating location methods with either teleseismic P phases or with regional phases. These two methods were separated during the development stage, because they present different problems in locating seismic events. However, since events are located with all available phases, the different versions must be combined and the techniques evaluated with all phases.

Two methods, LOCATION and SIMUL, were put together so that this program will compute locations with all phases. In this final program all regional phases use the simultaneous inversions method to modify Pn, Pg and Lg velocities during the iterative least squares cycles, and a matrix is inserted in the normal equation so that the correlation matrix method can be performed. If one chooses not to use the correlation matrix method, a unit matrix consisting of one's on the diagonal elements and zero's elsewhere is substituted for the correlation matrix, so there is no change in computing the solutions. Otherwise, non-zero coefficients will be assigned to each elements as described in section 3.2.

The non-diagonal elements of regional phases are all assigned zero values. This was necessary because we do not know proper values for them. However, correlation coefficients for regional phases should be determined in the future.

Six explosions were used to compare location errors without (NOMATX) and with a correlation matrix (MATRIX). Results are shown in Table VI. Location errors using either method are as high as those with teleseismic P only, see Table IV. Improvement of errors using the

MATRIX method as compared to the NOMATX method is about 4 kilometers from 26 to 22 kilometers. Using the correlation matrix method, only one event, GASBUGGY, resulted in inferior locations as compared to NOMATX.

Location errors given in Table VI average to greater than 20 kilometers. This was probably due to the high weighting factor imposed on teleseismic P. In Julian's (1973) location program, each station is weighted by the inverse of the assigned standard error. In the experiments shown in Table VI, standard errors are 0.2 for P, 1.0 for Pn and 3.0 for Pg and Lg. Consequently, P signals were weighted five times as large as those of Pn and 15 times as large as Pg or Lg. We think this weighting is not satisfactory. Although the values of standard errors for regional phases were adopted from McCowan and Needham (1978), the value for P should be about equal to the value of Pn. Results of weighted and unweighted runs shown in Table V indicate that locations with equal weighting for all regional phases are only slightly worse. This suggests that these weighting ratios needs to be further investigated. Note that using $\sigma = 0.2$ second for P leads to χ^2-error ellipses that do not include the true epicenters, so σ should be larger than 0.2 second for P.

In Table VII we show location errors of NOMATX and MATRIX, with equal weighting factors assigned to all phases. The improvement in locations are obvious. Furthermore, note that errors in MATRIX runs are much better than errors in NOMATRIX runs. From this result we conclude that equal weighting factors or more weighting for regional phases may be better. However, we emphasize that the mutual relations of weighting factors for various phases in the location program should be further investigated.

Depth estimates of these events are also shown in Table VI and VII. Depths computed with all phases are not much different from those depths computed with only teleseismic P (Table IV). This is expected because the *simultaneous inversions method does not estimate event depths*. The method computes the best fit linear slopes for Pn, Pg and Lg; but does not attempt to estimate the event depth. Consequently the computed depth using all phases will resemble the depth computed with P only. Improvements in depth estimates should be a research topic for the future.

True locations of these events relocated with the MATRIX method were all found to lie within the computed error ellipses. On the other hand, true locations of events relocated with the NOMATX method were mostly found outside of the computed error ellipses. When all phases were weighted equally in the NOMATX method, more true locations were found within the ellipses. This was the result of the better location obtained with the help of regional phases.

TABLE VI

Comparison of Location Errors, NOMATX vs. MATRIX
Using All Phases and Strong Weights on P

Event Name	No. P	No. Pn	No. Pg	No. Lg	Depth Restricted NOMATX	Depth Restricted MATRIX	Depth Free NOMATX (depth)	Depth Free MATRIX (depth)
FAULTLESS	106	5	5	5	6.89	5.77	* 5.36 (50.3)	* 5.73 (55.4)
RULISON	74	10	7	4	&12.36	&19.15	#14.97 (27.3)	#19.87 (11.2)
SHOAL	21	12	0	4	&44.32	28.02	*25.11 (102.8)	*18.05 (92.1)
GASBUGGY	60	12	12	6	&26.24	&37.33	#33.68 (39.2)	#38.52 (16.7)
SALMON	76	4	0	0	&22.71	12.86	#20.54 (-21.9)	12.39 (-23.8)
GNOME	44	18	1	0	&41.92	&30.02	#41.92 (-0.6)	#30.02 (-12.2)
Average					25.74	22.19	23.60	20.76

* - True location lies outside of the 3D 95% F-statistic ellipse

\# - True location lies outside of the 3D and 2D 95% F-statistic ellipses

& - True location lies outside of the 2D 95% F-statistic ellipse

TABLE VII

Comparison of Location Errors, NOMATX vs. MATRIX
Using All Phases and Equal Weights to All Phases

Event Name	Depth Restricted NOMATX	MATRIX	Depth Free NOMATX	(d)	MATRIX	(d)
FAULTLESS	3.37	1.47	* 2.38	(47.4)	* 0.78	(55.6)
RULISON	2.09	3.41	2.42	(17.9)	3.58	(5.8)
SHOAL	9.83	1.57	* 3.92	(86.7)	* 3.34	(82.4)
GASBUGGY	&10.11	& 9.86	# 9.59	(29.5)	$ 9.92	(3.7)
SALMON	&21.36	13.02	#19.13	(-22.0)	12.26	(-24.6)
GNOME	&25.50	10.77	#26.95	(-18.5)	11.36	(-4.0)
Average	12.04	6.68	10.73	(37.0)	6.87	(29.4)

* - True location lies outside of the 3D 95% F-statistic ellipse

\# - True location lies outside of the 3D and 2D 95% F-statistic ellipses

& - True location lies outside of the 2D 95% F-statistic ellipse

$ - True location lies inside of the 3D, but outside of the 2D 95% F-statistic ellipses.

5. CONCLUSIONS AND RECOMMENDATIONS

Seismic locations using teleseismic P data and using regional data present different problems. For this reason, our research in the previous years had different location programs for different phases. As a result we have developed many useful methods for different problems. The RELS location program on the VAX 11/780 at SDAC is a unified location program that can handle regional and/or teleseismic and/or back azimuth data. This program can be executed with or without the simultaneous inversions method, and with or without the correlation matrix method. Since all of these methods are independent programs on the TS44 system, the unified location program on the VAX is far more versatile than those separate programs on the TS44. A remaining task is to add the capability of using source and receiver models when the disk file structure for the VAX is agreed upon.

Three location methods, two using teleseismic data and one using regional phases, are evaluated by comparing location errors to the corresponding errors computed by the standard method. All methods showed some improvement over the standard method; and when all signals were combined, resultant location errors were about 6 kilometers on the average. Average errors with teleseismic P data alone are much larger, the best result being for the ACLOC3 method, with an average of about 14 kilometers. Locations with regional data gave better results, 6 kilometers on the average. These errors are approximately equal to those using all data.

Since the methods evaluated here do not use travel time corrections, these methods are not dependent on specially calibrated stations. These methods also do not require a priori knowledge of the local structures, therefore they can be used to locate seismic events in any parts of the world. Locations computed with these methods are absolute locations, not relative locations as are those using travel time corrections derived from a master event.

The correlation matrix method was not applied to the regional data, because correlation coefficients for Pn, Pg and Lg, obtained at proximate stations and the correlation coefficients for Pn and Pg, Pn and Lg, etc., observed at the same station are not known at the present. Determination of these coefficients, and also determination of standard errors to be used for weighting each input datum, should be investigated in the near future.

The RELS location program is also capable of using Lg arrivals for locating seismic events. In this study, all Lg arrivals were picked by analysts at SDAC at the beginning of the Lg wave train, rather than at the place where the maximum amplitude occurs. We also note that research on Lg has been concentrated on the attenuation phenomena, but not on the regional variations of Lg velocities. With the additional capability of locating events with Lg, the propagation phenomena of Lg in relation to the earth's heterogeneities should be further investigated.

Although the teleseismic locations examined here use a large number of observations, the accuracy of location with the teleseismic data is worse than the accuracy using a smaller number of regional data. There may be several explanations for this. The first reason is that $dT/d\Delta$ for regional phases are larger than those of teleseismic P, thus the least-squares method of fitting a standard travel time curve works better for regional phases.

Even so we are still surprised that one can obtain better location estimates by using data from a handful of regional stations rather than using a few dozens of teleseismic observations.

It is generally accepted that the Earth is more heterogeneous in the crust and upper mantle. If so, the effect of lateral heterogeneity would appear stronger on the travel time residuals of Pn and Pg as compared to those of teleseismic P. A brief review of true travel time residuals has showed however that the scatter of residuals is much larger for teleseismic P than for Pn and Pg. Since true residuals do not include the effect of source mislocation, these values reflect true structural heterogeneities at various depths. A larger scatter in P suggests that the Earth is also very heterogeneous in the lower mantle. We will continue to investigate this in the future.

ACKNOWLEDGEMENTS

The Lg arrivals of the nuclear explosions used here were determined by Esther Gordon, Peggy Marshall and Robert Wagner. John Lambert assisted in converting location programs from TS44 to VAX 11/780.

REFERENCES

Chang, A. C., R. H. Shumway, R. R. Blandford and B. W. Barker (1980). Two methods to improve location estimates - Preliminary results. AL-80-3, Teledyne Geotech, Alexandria, Virginia 22314.

Chang, A. C., D. W. Rivers and J. A. Burnetti (1981a). Improvements in seismic locations and error ellipses. AL-81-1, Teledyne Geotech, Alexandria, Virginia 22314.

Chang, A. C., D. W. Rivers and J. A. Burnetti (1981b). Improved Location with regional events. VSC-TR-81-10, Teledyne Geotech, Alexandria, Virginia 22314.

Cleary, J. and A. L. Hales (1966). An analysis of the travel times of P-waves to North American stations in the distance range of 32 to 100 . Bull. Seism. Sos. Am., 56, 467-489.

Flinn, E. A., E. R. Engdahl and A. R. Hill (1974). Seismic and Geographical Regionalization. Bull. Seism. Soc. Am., 65, 771-993.

Julian, B. R. (1973). Extension of standard event location procedures, Seismic Discrimination SATS, Lincoln Laboratory, M.I.T., 30 June 1973, 4-9.

McCowan, D. W. and R. E. Needham (1978). The use of crustal phases in locating explosion epicenters. Seismic Discrimination SATS, Lincoln Laboratory, M.I.T., 9 June 1978, 1-2.

Poupinet, G. (1976). Sur l'existence d'un manteau a teres faible vitesse sous les Alps occidentals et ses implications tectoniques. Bull. Soc. geol. Fr., 5, 1073-1083.

Sengupta, M. K. and B. R. Julian (1976). P-wave travel times from deep earthquakes. Bull. Seism. Soc. Am., 66, 1555-1579.

Veith, K. F. (1975). Refined hypocenters and accurate reliability estimates. Bull. Seism. Soc. Am., 65, 1199-1222.

APPENDIX I

LOCATION ERROR ELLIPSES FOR RELS

LOCATION ERROR ELLIPSES FOR RELS

This is a brief note which attempts to clear up certain recurring problems in our assigning confidence limits to event locations. The problems recur because we haven't adequately documented the algorithm, so there is always uncertainty and/or faulty memory about exactly what it does. This note will sketch the essentials of the process as it stands now, so that the necessary changes can be made to the routine in order to adjust its computation to those which are required by RELS.

For an initial location estimate $\vec{x}_o$, the arrival time (and back azimuth) residuals are expanded as

$$\vec{\delta}_t = \underset{\sim}{B} (\vec{x} - \vec{x}_o) + \vec{\varepsilon} \tag{A-1}$$

where we assume that

$$\mathrm{cov}(\vec{\varepsilon}) = \langle \varepsilon\varepsilon' \rangle = \sigma^2 \underset{\sim}{\Sigma} = \sigma^2 \begin{pmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \dots & \sigma_N^2 \end{pmatrix} \tag{A-2}$$

ordinarily, although our new program doesn't require that $\underset{\sim}{\Sigma}$ be diagonal. If we have chosen the weight $w_i = 1/\sigma_i^2$ to be the true variance of the i^{th} observation, then the scale factor $\sigma^2 = 1$. The least-sqaures solution to the linearized equation is

$$\hat{\vec{x}} = \vec{x}_o + (\underset{\sim}{B}^T \underset{\sim}{\Sigma}^{-1} \underset{\sim}{B})^{-1} \underset{\sim}{B}^T \underset{\sim}{\Sigma}^{-1} \vec{\delta}_t. \tag{A-3}$$

We see that back azimuth and arrival time residual may both appear as components of $\vec{\delta}_t$ even though they don't have the same units, since they are weighted by their respective a priori variances so that all the products in the matrix multiplication have units of km. If we assume that the least-squares estimate $\hat{\vec{x}}$ has a mutivariate Gaussian distribution about the true location $\vec{x}$, then

$$(\vec{x} - \hat{\vec{x}})' [\sigma^2 (\underset{\sim}{B}^T \underset{\sim}{\Sigma}^{-1} \underset{\sim}{B})^{-1}]^{-1} (\vec{x} - \hat{\vec{x}}) \le \chi^2_{q:\alpha} \tag{A-4}$$

is the confidence region at the $1-\alpha$ level (we choose $\alpha = 5\%$), or

$$(\vec{x} - \hat{\vec{x}})' (B^T \Sigma^{-1} B) (\vec{x} - \hat{\vec{x}}) \le \sigma^2 \chi^2_{q:\alpha}. \tag{A-5}$$

Here q is the number of dimensions of $\vec{x}$. For the conventional location algorithm, q is 3 or 4, depending on whether it is run in the depth-free or depth-restrained mode. For SIMUL, however, $q \leq 10$, since we now determine coefficients a_{Pg}, b_{Pg}, a_{Pn}, etc., which describe the slopes and intercepts for regional phase travel-time relations determined a posteriori (Chang et al., 1981b). If we do not believe our a priori estimate $\sigma^2 = 1$, we replace it a posteriori with the calculated variance

$$\hat{\sigma}^2 = \frac{1}{N-q} [\vec{\delta}_t - \underset{\sim}{B}(\hat{\vec{x}} - \vec{x}_o)]' \underset{\sim}{\Sigma}^{-1} [\vec{\delta}_t - \underset{\sim}{B} (\hat{\vec{x}} - \vec{x}_o)], \tag{A-6}$$

where, for the final iteration, $\vec{x}_o = \hat{\vec{x}}$, so

$$\hat{\sigma}^2 = \frac{1}{N-q} \vec{\delta}_t' \underset{\sim}{\Sigma}^{-1} \vec{\delta}_t. \tag{A-7}$$

Note that we still assume that our initial assignation of the relative weights σ_i^2/σ_j^2 is correct; obviously, the computed location $\hat{x}$ as well as the confidence region will change if we decide to weight P_n the same as P, for example. Now $\hat{\sigma}^2 (N-q)/\sigma^2$ has the χ^2 distribution with N-q degrees of freedom, so the ratio of the previous χ_q^2 statistic, divided by q degrees of freedom, to this one, divided by N-q, has the F-distribution with q, N-q degrees of freedom:

$$\frac{[(x - x)'[\sigma^2 (\underset{\sim}{B}^T \underset{\sim}{\Sigma}^{-1} \underset{\sim}{B})^{-1}]^{-1} (x - x)']/q}{[\hat{\sigma}^2 (N-q)/\sigma^2]/(N-q)} \leq F_{q,N-q:\alpha}, \tag{A-8}$$

and the confidence region becomes

$$(\vec{x} - \hat{\vec{x}})'(\underset{\sim}{B}^T \underset{\sim}{\Sigma}^{-1} \underset{\sim}{B}) (\vec{x} - \hat{\vec{x}})' \leq q \hat{\sigma}^2 F_{q,N-q:\alpha} \tag{A-9}$$

The confusion arises when we want to consider spaces of dimensionality less than q. For an m-dimensional subspace we replace $\vec{x}$ and $\hat{\vec{x}}$ by $\vec{z}$ and $\hat{\vec{z}}$, their projections onto the subspace. (Since the components of $\vec{x}$ are $\vec{x} = [x_y, x_x, x_t, x_h, x_{aPn}, \ldots, x_{bLg}]$ where + y = north, + x = east, + t = future, and + h = down, we will conventionally perform this projection into 3- and 2-dimensional space by simply dropping the final 7 then 8 components. We can, of course, consider 4 (or higher) -dimensional confidence regions, as was done in the initial version of LOCATION by Julian (1973), but their interpretation is difficult to grasp, and it will be better to consider the uncertainties δ_t, δ_{aPn} etc., in terms of the standard errors, as we shall show later.) Let us define

$$\underset{\sim}{S}^{-1} \equiv \underset{\sim}{B}^T \underset{\sim}{\Sigma}^{-1} \underset{\sim}{B} \tag{A-10}$$

so that $\underset{\sim}{S} = (\underset{\sim}{B}^T \underset{\sim}{\Sigma}^{-1} \underset{\sim}{B})^{-1}$ is a qxq matrix. We form an mxm submatrix $\underset{\sim}{E}$ by setting

$$E_{ij} = S_{ij} \text{ for } i = 1,\ldots,m \text{ and } j = 1,\ldots,m.$$

Our q-dimensional chi-square confidence region reduces to

$$(\vec{z} - \hat{\vec{z}})' \, [\sigma^2 \underset{\sim}{E}]^{-1} \, (\vec{z} - \hat{\vec{z}}) \leq \chi^2_{m:\alpha.} \tag{A-11}$$

Note that although $\underset{\sim}{E}$ is a submatrix of $\underset{\sim}{S}$, $\underset{\sim}{E}^{-1}$ is <u>not</u> a submatrix of $\underset{\sim}{S}^{-1} = \underset{\sim}{B}^T \underset{\sim}{\Sigma}^{-1} \underset{\sim}{B}$. Watch out for this when refering to Flinn's classic article on error ellipses (1965), which has problems with this distinction, as does the "corrections" article published years later. Now no changes are to be made in the calculation of $\hat{\sigma}^2$; it still has N-q degrees of freedom. The F-stataistic confidence region in m-dimensions is thus

$$(\vec{z} - \hat{\vec{z}})' \, E^{-1} \, (\vec{z} - \hat{\vec{z}}) \leq m \, \hat{\sigma}^2 \, F_{m,N-q:\alpha.} \tag{A-12}$$

If we know the true location, as in the case for NTS shots, we can now see whether this confidence region around the computed hypocenter (m=3) or epicenter (m=2) does in fact include the true location. Since $\hat{\sigma}^2$ is computed by the program and is stored as variable "EMS" in common block LOKCOM, and Since $\underset{\sim}{S}^{-1}$ is passed to the error ellipse subroutine DMHLP via the argument list, we can simply plug the appropriate values into this formula and see whether the left-hand side is indeed smaller than the right-hand side. (The value of the right-hand sidee is computed automatically by DMHLP; it is nno longer necessary for the programmer to input the appropriate value of $F_{m,N-q}$.) We can also now determine the relationship between the 2-d error ellipse and the 2-d projection of the 3-d error ellipsoid. Although $\underset{\sim}{E}_{2-d}$ is a submatrix of $\underset{\sim}{E}_{3-d}$, $\underset{\sim}{E}^{-1}{}_{2-d}$ is not a submatrix of $\underset{\sim}{E}^{-1}{}_{3-d}$, so the 2-d ellipse differs not only in size (right-hand side of the inequality) but also in shape (left-hand side) from the 2-d projection off the error ellipsoid (i.e., a horizontal cross-section of the vertical cyclinder containing it). [Note that these are both different from yet a third 2-d "error ellipse" of sorts, namely the intersection of the 3-d ellipsoid with the surface plane h = 0.] It is thus possible that the true hypocenter lies within the error ellipsoid even though the true epicenter lies outside the error ellipse, or vice versa. As an example of the first situation, we note that if the calculated hypocenter lies directly below the true hypocenter but at a much greater depth, then the true hypocenter would probably lie outside (namely, above) the 3-d error ellipsoid, but the true epicenter would be in the exact center of the 2-d error ellipse. As an example of the opposite situation, we consider the special case in which one of the principal (i.e., symmetry) axes of the 3-d error ellipsoid is vertical. The 3x3 matrix $\underset{\sim}{E}_{3-d}$ may then be written as

$$\underset{\sim}{E}_{3-d} = \left(\begin{array}{cc:c} \multicolumn{2}{c:}{\multirow{2}{*}{$\underset{\sim}{E}_{2-d}$}} & 0 \\ & & 0 \\ \hdashline 0 & 0 & e_{33} \end{array} \right), \qquad \text{(A-13)}$$

so we have

$$\underset{\sim}{E}^{-1}{}_{3-d} = \left(\begin{array}{cc:c} \multicolumn{2}{c:}{\multirow{2}{*}{$\underset{\sim}{E}^{-1}_{2-d}$}} & 0 \\ & & 0 \\ \hdashline 0 & 0 & 1/e_{33} \end{array} \right). \qquad \text{(A-14)}$$

All that this says is that the <u>shape</u> of the 2-d projection of a vertically symmetric 3-d error ellipsoid is the same as that of the 2-d ellipse, a result which one could have guessed geometrically and which clearly is approximately valid even if the 3-d ellipsoid is tilted slightly from the vertical, i.e., if $|e_{13}| << |e_{33}|$ and $|e_{23}| << |e_{33}|$. If the computed hypocenter and the true hypocenter are at the same depth, we see that the quantity $(\vec{Z}-\vec{Z})' \underset{\sim}{E}^{-1} (\vec{Z}-\vec{Z})$ is the same for the 3-d and the 2-d cases. The <u>size</u> of the 2-d projection of the 3-d ellipsoid differs from that of the 2-d ellipse, however; in fact, the ratio of their areas is given by $3\ F_{3,N-q}/2F_{2,N-q} = 1.3$ for $N \to \infty$. In this special case, then, the true hypocenter will lie within the 3-d error ellipsoid but the true epicenter will lie outside the 2-d error ellipse if $1.0 < (\vec{Z}-\vec{Z})' \underset{\sim}{E}^{-1} (\vec{Z}-\vec{Z})/(2F_{2,N-q}\ \hat{\sigma}^2) < 1.3$.

We can now compute the lengths of the axes of the m-dimensional error ellipsoid and the spatial orientations of these axes. In order to do so, we rotate from the fixed y-, x-, and h-axes of space to the symmetry axes of the error ellipsoid. In this new corrdinate system the matrix $\underset{\sim}{E}^{-1}$ becomes

$$\underset{\sim}{\Lambda} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \qquad \text{(A-15)}$$

where λ_1, λ_2, λ_3 are the eigenvalues of $\underset{\sim}{E}^{-1}$. The basis vectors of this rotated coordinate system are of course the eigenvectors of $\underset{\sim}{E}^{-1}$; since they are orthonormal, their components (expressed in the y-, x-, h-space) are their direction cosines with respect to the fixed axes. In the rotated coordinate system, which we denote as ζ_1, ζ_2, ζ_3, the 3-d error ellipsoid is

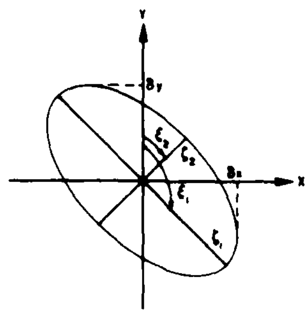
$$\lambda_1 \zeta_1^2 + \zeta_2^2 + \lambda_3\ \zeta_3^2 \leq 3\hat{\sigma}^2 F_{3,N-q:\alpha} \qquad \text{(A-16}$$

with a similar equation for the 2-d error ellipse, where we have now simplified matters by setting the origin of the coordinate system at the computed location $\hat{Z}$. The length of the ζ_1-axis of the ellipsoid is given by setting ζ_2 and ζ_3 equal to zero in this expression:

$$\zeta_1 = \sqrt{3\hat{\sigma}^2 F_{3,N-q:\alpha}/\lambda_1} \qquad \text{(A-17)}$$

and similarly for the lengths of the ζ_2- and ζ_3-axes. We can thus find the lengths and orientations of the error ellipsoid axes by diagonalizing $\underset{\sim}{E}^{-1}$; our program uses the subroutine EIGEN to do this. The SDL report on HYPO (Flinn, 1965b) indicates that a geometrical manipulation is used in that program to accomplish this, but diagonalizing $\underset{\sim}{E}^{-1}$ seems to be more straightforward, especially in the 3-d case.

Even though we now know the lengths of the ellipsoid axes, we still do not know the maximum range (at the $1-\alpha$ confidence level) in each direction of the possible values of $\vec{x}$. For example, in the diagram below, we know the lengths of the semi-major and semi-minor axes ζ_1 and ζ_2 and their azimuths ζ_1 and ζ_2, but we have not yet determined δy and δx. (Figure 1):



For the moment let us scale the ellipse to a unit size by replacing the right-hand side of its equation by unity:

$$(\vec{z} - \hat{z})' \underset{\sim}{E}^{-1} (\vec{z} - \hat{z}) = 1 \qquad \text{(A-18)}$$

which we expand as

$$b_{11} y^2 + 2b_{12}\, xy + b_{22}\, x^2 = 1 \qquad \text{(A-19)}$$

where $\underset{\sim}{E}^{-1} = \begin{pmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{pmatrix}$ Differentiating with respect to x, we obtain

$$2\, b_{11}\, yy' + 2\, b_{12}\, y + 2\, b_{12}\, xy' + 2\, b_{22}\, x = 0 \tag{A-20}$$

where $y' = \frac{dy}{dx}$. At the maximum extension in the y-direction of the ellipse δy, we have $y'_{\delta y} = 0$, so

$$2\, b_{12}\, y_{\delta y} + 2\, b_{22}\, x_{\delta y} = 0$$

or

$$x_{\delta y} = (-b_{12}/b_{22}) y_{\delta y}. \tag{A-21}$$

Substituting into the equation for the ellipse, we have

$$b_{11}\, y^2_{\delta y} - -2(b^2_{12}/b_{22})\, y^2_{\delta y} + (b^2_{12}/b_{22})\, y^2_{\delta y} = 1 \tag{A-22}$$

$$(b_{11} - b^2_{12}/b_{22})\, y^2_{\delta y} = 1$$

$$y_{\delta y} = 1/\sqrt{b_{11} - b^2_{12}/b_{22}} \quad . \tag{A-23}$$

The maximum northward extent of the error ellipse is thus given in terms of $\underset{\sim}{E}^{-1}$; a similar analysis can be used to determine δx or, in the 3-d case, δh.

A much faster approach involves using the matrix $\underset{\sim}{S} = (\underset{\sim}{B}^T \Sigma^{-1} \underset{\sim}{B})^{-1}$ rather than the matrix $\underset{\sim}{E}^{-1}$. To see this, consider the case in which the submatrix $\underset{\sim}{E}$ is in fact the entire matrix $\underset{\sim}{S}$. (This would happen if the location were performed with all variables restrained except for x and y.) If $\underset{\sim}{S} = \begin{pmatrix} a_{11} & a_{12} \\ a_{12} & a_{22} \end{pmatrix}$, then

$$\underset{\sim}{E}^{-1} = \underset{\sim}{S}^{-1} = \frac{1}{a_{11}\, a_{22} - a^2_{12}} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{12} & a_{22} \end{pmatrix} \equiv \begin{pmatrix} b_{11} & b_{12} \\ b_{12} & b_{22} \end{pmatrix}. \tag{A-24}$$

$$
\begin{aligned}
\text{Then} \qquad \delta v &= 1/\sqrt{b_{11} - b_{12}^2/b_{22}} \\
&= \sqrt{b_{22}} \,/\sqrt{b_{11}\, b_{22} - b_{12}^2} \\
&= \sqrt{a_{11}} \,/\Big[\frac{1}{\sqrt{a_{11}\, a_{22} - a_{12}^2}} \;\; \sqrt{a_{11}\, a_{22} - a_{12}^2}\Big] \\
&= \sqrt{a_{11}}
\end{aligned}
\qquad \text{(A-25)}
$$

In general

$$
\delta x_i = \{[\mathrm{cov}\ (\hat{\vec{x}})]_{ii}\}^{1/2} \qquad \text{(A-26)}
$$

where $\mathrm{cov}(\hat{\vec{x}}) = \sigma^2 \ (\underset{\sim}{B}^T \ \underset{\sim}{\Sigma}^{-1} \ \underset{\sim}{B})^{-1}$. The maximum extents in each direction of the unit error ellipse (the "standard errors") are thus simply the square roots of the diagonal elements of $\underset{\sim}{S}$. We must now scale these values by replacing unity on the right-hand side of the defining equations by some constant K; the standard error will then be multiplied by the factor $\sqrt{K}$. In our analyses of the 2-d and 3-d confidence regions we have used $K = 2\,\hat{\sigma}^2 F_{2,N-q:\alpha}$ and $K = 3\hat{\sigma}^2 F_{3,N-q:\alpha}$. As we have pointed out, this depends on the dimensionality of the confidence region; for the 3-d error ellipse, the maximum ranges δx and δy were shown to be $\sqrt{1.3}$ times larger than δx and δy for the 2-d error ellipse in a special case. The best way of treating the uncertainty in a single variable is to regard it simply as a one-dimensional confidence ellipsoid. If we assume that the variance is known <u>a priori</u>, we thus have

$$
\begin{aligned}
\delta v &= \{[\mathrm{cov}\ (\hat{\vec{x}})]_{11}\}^{1/2} \cdot \sqrt{\sigma^2 \ \chi^2_{1:\alpha}} \\
&= \{[(B^T \ \Sigma^{-1} \ B)^{-1}]_{11}\}^{1/2} \cdot 1.960.
\end{aligned}
\qquad \text{(A-27)}
$$

(if $\sigma^2 = 1$, which we define it to.) This is of course just what you would expect for the 95% confidence region, namely that it is 1.96 standard deviations. Note that this <u>a priori</u> estimate depends solely on the geometry of the stations and the hypocenter, and it does not depend on the measured arrival times. We are therefore more interested in the *a posteriori* estimate

$$\delta y = \{[\operatorname{cov}\,(\hat{x})]_{11}\}^{1/2} \cdot \sqrt{1\hat{\sigma}^2 F_{1,N-q:\alpha}}$$

$$= \{[\underset{\sim}{B}^T \underset{\sim}{\Sigma}^{-1} \underset{\sim}{B})^{-1}]_{11}\}^{1/2} \hat{\sigma} \sqrt{F_{1,N-q:\alpha.}} \qquad \text{(A-28)}$$

This is the best way to handle the uncertainties δh and δt. The program currently calculates the standard errors and the variances $\hat{\sigma}^2$, so it can trivially be adjusted to compute δ h and δt in this manner. We propose that the output of the RELS lcoation error subroutine be:

$\hat{\sigma}^2$	the *a posteriori* variance
$\zeta_{1(2-d)}$, $\zeta_{2(2-d)}$	the lengths of the semi-major and semi-minor axes of the confidence ellipse
$\xi_{1(2-d)}$, $\xi_{2(2-d)}$	the azimuths (in radians clockwise from north?) of these axes
δh, δt	the standard errors in depth and origin time, multiplied by the appropriate factors.

The parameters $\zeta_{1(2-d)}$, $\zeta_{2(2-d)}$, $\delta h_{(1-d)}$ and $\delta t_{(1-d)}$ can be evaluated for χ^2 and/or F-statistic cases. Optional parameters to return are $\zeta_{1(1-d)}$; $\zeta_{2(3-d)}$, $\zeta_{3(3-d)}$, $A = \pi \cdot \zeta_{1(2-d)}\ \zeta_{2(2-d)}$, and $V = \cdot\pi\ \zeta_{1(3-d)} \cdot \zeta_{2(3-d)} \cdot \zeta_{3(3-d)}$ in the χ^2 and/or F-statistic cases. We recommend that the angles ξ be expressed relative to the fixed y-, x-, and h- axes rather than as Euler angles within the ellipsoids, as was done by Julian (1973).

END

DATE
FILMED
8-82

DTIC